**Chapter # 05 (Exponential and Logarithmic Function)**

**5.2 One-to-One Function; Inverse Functions:**

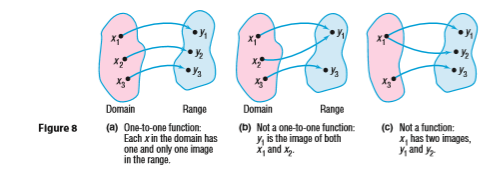
**Objective:** 1. Determine Whether a Function Is One-to-One

2. Determine the Inverse of a Function Defined by a Map or a Set of Ordered Pairs

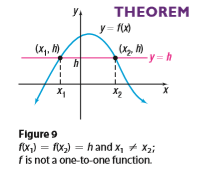
3. Obtain the Graph of the Inverse Function from the Graph of the Function

4. Find the Inverse of a Function Defined by an Equation

**One-to-One Function:** A function *f* is one-to-one if any two different inputs in the domain correspond to two different outputs in the range. That is, if are two different inputs of a function *f*, then *f* is one-to-one if . Another way, a function *f* is one-to-one if no *y* in the range is the image of more than one *x* in the domain. A function is not one-to-one if any two (or more) different elements in the domain correspond to the same element in the range.



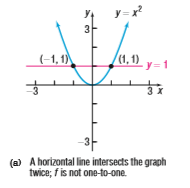
**Theorem (Horizontal-line Test):** If every horizontal line intersects the graph of a function f in at most one point, then f is one-to-one.



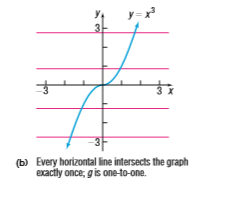
**Question:** Using the horizontal-line test, to determine whether the following functionis one-to-one

(a) (b)

**Solution: (a)** The following figure illustrates the horizontal-line test for . The horizontal line intersects the graph of *f* twice, at and , so *f* is not one-to-one



**(b)**The following figure illustrates the horizontal-line test for . Because every horizontal line intersects the graph of *g* exactly once, it follows that *g* is one-to-one.



**Theorem:** A function that is increasing on an interval I is a one-to-one function on I. A function that is decreasing on an interval I is a one-to-one function on I.

**Inverse Function:** Suppose that *f* is a one-to-one function. Then, corresponding to each *x* in the domain of *f*, there is exactly one *y* in the range (because *f* is a function); and corresponding to each *y* in the range of *f*, there is exactly one *x* in the domain (because *f* is one-to-one). The correspondence from the range of *f* back to the domain of *f* is called the inverse function of *f* . The symbol is used to denote the inverse function of *f*.

**Example:** Find the inverse of the function defined by the map. Let the domain of the function represent certain states, and let the range represent the states’ populations (in millions). Find the domain and the range of the inverse function.

|  |  |
| --- | --- |
| State | Population |
| Indiana | 6.5 million |
| Washington | 6.1 million |
| South Dakota | 0.8 million |
| North Carolina | 9.8 million |
| Tennessee | 3.8 million |

**Solution:**The function is one-to-one. To find the inverse function, interchange the elements in the domain with the elements in the range. For example, the function receives as input Indiana and outputs 6.5 million. So the inverse receives as input 6.5 million and outputs Indiana. The inverse function is shown here.

|  |  |
| --- | --- |
| Population | State |
| 6.5 million | Indiana |
| 6.1 million | Washington |
| 0.8 million | South Dakota |
| 9.8 million | North Carolina |
| 3.8 million | Tennessee |

The domain of the inverse function is . The range of the inverse function is {Indiana, Washington, South Dakota, North Carolina, Tennessee}. If the function *f* is a set of ordered pairs , then the inverse function of *f*, denoted , is the set of ordered pairs .

**Question:** Find the inverse of the following one-to-one function:

State the domain and range of the function and its inverse.

**Solution:**The inverse of the given function is found by interchanging the entries in each ordered pair and so is given by

The domain of the function is {-3, -2, -1, 0, 1, 2, 3}. The range of the function is {-27, -8, -1, 0, 1, 8, 27}.

The domain of the inverse function is {-27, -8, -1, 0, 1, 8, 27}. The range of the inverse function is

{-3, -2, -1, 0, 1, 2, 3}

**Note That:** Domain of *f* = Range of and Range of *f* = Domain of

, where *x* is the domain of *f*

, where x is the domain of

**Question:** (a) Verify that the inverse of is

(b) Verify that the inverse of is

**Solution: (a)** , for all *x* in the domain *g*

and , for all x in the domain of Verified.

**(b)** , for all *x* in the domain of *f*

and , for all x in the domain of

Verified.

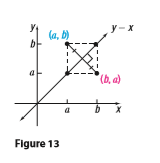
**Question:** Verify that the inverse of is . For what values of *x*is ? For what values of *x* is

**Solution:** The domain of *f* is and the domain of is

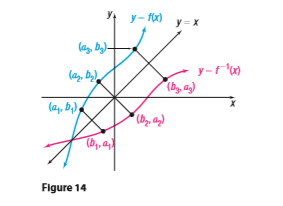
Now, , provided

, provided

**Obtain the Graph of the Inverse Function from the Graph of the Function:** Suppose that is a point on the graph of a one-to-one function *f* defined by . Then . This means that , so is a point on the graph of the inverse function . The relationship between the point on *f* and the point on is shown in the following figure.



**Theorem:** The graph of a one-to-one function *f* and the graph of its inverse function are symmetric with respect to the line y = x.



**How to Find the Inverse Function:** Step-by-Step

**Step1:**Replace with y. In , interchange the variables *x* and *y* to obtain . This equation defines the inverse function implicitly

**Step2:** If possible, solve the implicit equation for *y* in terms of *x* to obtain the explicit form of ,

**Step3:** Check the result by showing that and

**Question:** The function, is one-to-one. Find its inverse function and check the result.

**Solution:**

**Step 1:** Replace with *y* and interchange the variables *x* and *y* in

to obtain

**Step 2:** Solve for *y*

The inverse function is

**Step3:** Check

and Verified.

**H. W: Exercise 5.2: Problem No. 21-44, 51-70 and 91 - 94**

**Exercise 5.2:**

**Question no. 21 - 26 are same( Using Horizontal line test to determine one-to-one function)**

**Question no. 27 - 34 are same:**

**Question no. 35 - 44 are same:**

**Question37:** Verify that the functions *f* and *g* are inverses of each other by showing that . Give any values of *x* that need to be excluded from the domain of *f* and the domain of *g*. Where and

**Solution:**

and

Thus *f* and *g* are inverses to each other.

**Question no. 51 - 74 are same:**

**Question53.** Given,

(a) Find its inverse function and check your answer

(b) Find the domain and range of

(c) Graph on the same coordinate system

**Solution:** Given

(a) Consider

Now interchange the variables *x* and *y* to obtain

Ans.

**Verification:**

and Verified

(b) Domain of f = Range of = All real numbers

Range of f = Domain of = All real numbers